# Theories Regarding Risk Rates: Structure, Factors That Influence Fixes Income Financial Investments 

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#### Abstract

Bonds - as a representative type of securities for fixed income financial investments with a long-term maturity - have a price which reflects the disadvantages of interest rate modifications. This price illustrates a well-known characteristic of financial markets, respectively: the high volatility of long-term bonds rate in comparison with short-term securities rate. The variations of interest rates reflect the risks of investments made in long-term bonds. Investors and financial investment managers are permanently concerned with protecting against interest rate risk.


Keywords: default risk, anticipation theory, the theory of the segmented markets, liquidity premium theory, preferred areas theory.

## 1. Introduction

Long-term fixed income financial investments pose an interest rate risk that is higher in comparison with short-term securities rate. The duration of owning securities which have an equal maturity is different; thus, because the final value of securities is fixed, this value cannot be affected by interest rate variations. The return for owning such securities is equal with the actuarial rate calculated at its acquisition price.

Buying high rate bonds is the only important choice for a good placement. Actually, everything depends on the interest rate calculated during the period of owning the bonds. The correct measurement of the income for owning bonds during a certain period represents their return, i.e. the return rate. Return is the sum of payments made during the owning period and it is a capital income that finally is gained (including by reimbursement) and it is calculated in relation to the initial price.

Return results from owning bonds and it can be different from their interest rate. Return results after a placement is made and it is calculated in relation to the evolution of the interest actuarial rate, which may amount at the initial level of an owned asset until this asset reaches its maturity

## 2. Literature review

The rate structure expressed as an interest rate risk. This rate structure is explained in relation to three factors:
(a) Non-payment risk (default risk)

Non-payment risk (or default risk) is one of the characteristics that are specific to bonds which are influenced by the interest rate; in other words, the bond issuer is unable to pay interests or to reimburse the main bond owner.

In general it is considered that Treasury bonds do not have a nonpayment risk in comparison with the ones issued by enterprises because governments can increase charges for debt payment or they can issue currency for paying debts. These bonds are known as riskless bonds. Similarly, in reach countries State debts are considered risky.

The difference between the interest rate of risky bonds and riskless bonds is known as premium risk. It represents the additional rate that the owner of a bond gets and it poses a nonpayment risk for accepting the owning of more riskless bonds.

The analysis of supply and demand existing on bonds market allows us to explain why a bond that poses a nonpayment risk still pays a positive risk premium and why this premium increases alongside with nonpayment risk.

For evaluating the effect of non-payment risk on the interest rate, we have elaborated the charts of supply and demand for riskless bonds issued by Treasury and for the private bonds issued by an enterprise - see picture 1. We presume that private bonds have the same risk as the ones issued by the Treasury. Their price and their interest rate are initially equal ( $P_{1}^{C}=P_{1}^{T}$ and $i_{1}^{C}=i_{1}^{T}$ ) naturally if the risk premium of private bonds $\left(i_{1}^{C}-i_{1}^{T}\right)$ is null.

Figure 1. The effects of an increase in nonpayment risk for bonds within an enterprise


An increase of the nonpayment risk for private bonds modifies the demand curve for these bonds,
 " $\mathrm{D}_{2}{ }^{\mathrm{T}}$ ". The balance prize (on the left axis) for private bonds named " $\mathrm{P}_{1}{ }^{\mathrm{c}}$ " and " $\mathrm{P}_{2}{ }^{\mathrm{c}}$ " and the interest rate for these bonds, named " $\mathrm{i}_{1}{ }^{\mathrm{c}}$ " and " $\mathrm{i}_{2}{ }^{\mathrm{c}}$ " (on the vertical axis). On the Treasury bonds market one can find the balance price " $\mathrm{P}_{1}{ }^{\mathrm{T}}$ " and " $\mathrm{P}_{2}{ }^{\mathrm{T}}$ " and the interest rate " $\mathrm{i}_{1}{ }^{\mathrm{T}}$ " and " $\mathrm{i}_{2}{ }^{\mathrm{T}}$ ". The difference between " $\mathrm{i}_{2}{ }^{\mathrm{T}}$ and " $\mathrm{i}_{2}{ }^{\mathrm{c} \text { " }}$ identifies the risk premium that is specific to private bonds.

If nonpayment is more likely to appear, for example because the enterprise suffers losses, then the nonpayment risk is increased and this reduces the anticipated return of the bonds. Moreover, return is uncertain. Active demand theory stipulates that if the anticipated return of an asset increases in relation to other assets (in this case we refer to the riskless bonds issued by the Treasury) or if relative risk increases, its demand is reduced.
The demand curve for private bonds moves to the left in part (a) - Figure 1.

## (b) Liquidity

The second characteristic of bonds, which affects the interest rate, is its liquidity. A liquid active is an asset which can be converted into money quickly and at a low price. Furthermore, an asset is liquid when its owning is desirable and any choice from the existing assets seem to be similar. In several countries, the Treasury assets are long-term bonds and are more liquid because modifications are quite large, while these assets are facile and can be sold quickly. Private bonds are, in general, less liquid because no enterprise issues bonds as the State does. They can be issued for sale in emergency and it might be difficult to find buyers immediately.

The supply and demand analysis of bonds reveals that liquidity is affected by the interest rate. If we suppose from the beginning that private bonds and the Treasury bonds have the same liquidity and are identical from all the other points of view, their price and the balance interest rate are similar. If private bonds become less liquid, their demand drops, as well as the price, while the interest rate goes up. Similarly, the Treasury bonds supply increases because their liquidity increases in relation to private bonds, their price goes up and the interest rate decreases.

At the same time, "the premium risk" of private bonds and Treasury bonds does not represent the only nonpayment risk difference; the same is true for the liquidity difference between the two. This premium should
be named "risk and liquidity premium", even though the name of this premium remains unchanged out of habit and because it is more facile to call it this way.

## (c) Fiscality

The behavior of municipal bonds is mysterious, especially in the USA. Bonds issued by local bodies are less liquid than the ones issued by the American Treasury although for more than 60 years the interest rate has been lower than the one for Treasury bonds. This happens in the countries in which the bonds of local bodies are exempted from the federal taxes on income and this increases their return. After 25 years the USA has reconsidered federal taxes.

## 3. The rate structure through maturity of the interest rate

Another essential characteristic of bonds, which affects their interest rate, apart from the identification of their structure through the interest rate risk, is represented by the maturity of the interest rate: two bonds whose risk, liquidity and fiscality are identical may have different interest rates because their maturity is different. The representation of interest rates for bonds in relation to their maturity is known as the yield curve.

The yield curve is the structure identified through the interest rate for a category of bonds, for example for the Treasury bonds.

The yield curve can be descending, flat or ascending (we refer to the inverted yield curve). It is ascending when the long-term interest rate is higher than short-term interest rates; it is flat when these rates are identical and it is descending when short-term interest rates are higher. Rate curves may be found in more complicated shapes, which could be successively ascending and descending or vices versa.

A good theory of the structure through the interest rate maturity is not the only one which can explain the varied shapes that rate curves can take; however, similarly, the following facts have been proved valid by many empirical studies:
a) The interest rate for different bonds which have various maturities varies, in general, in time.
b) When the short-term interest rate is low, rate curves are more likely to be ascending because they are high.
c) Rate curves are normally ascending.

There are three theories that explain the structure through the maturity of the interest rate:
A. Anticipation theory
B. Segmented markets theory
C. Liquidity premium theory.

Anticipation theory explains the first two statistical facts except for the third one. The theory of segmented markets explains the last statistical fact but it fails to explain the first two ones. The theory of the liquidity premium is a synthesis of the first two ones, which explicitly combines all the three facts.

### 3.1. Anticipation theory

According to the anticipation theory - in connection with the structure explained through the interest rate maturity - the interest rate of a long-term bond is equal with the average of the short-term interest rate for the economic agents selected to notice the evolution of bonds in time. For example, if we anticipate that the short-term interest rate maturity corresponds with the $10 \%$ average for a period of 5 successive years, the anticipation theory foresees that the interest rate for the 5 -year bonds will be equal with $10 \%$. If we anticipate that the short-term interest rate increases after 5 years and that the average for 20 successive years amounts at $11 \%$, the interest rate for a 20 -year bond will be $11 \%$, i.e higher than the interest rate for 5 -year bonds.

According to this theory the interest rate of bonds with different maturities differs from the short-term interest rate with anticipated maturity for different periods in the near future.

The final hypothesis of this theory is that the buyer does not prefer to buy bonds which have a higher maturity than the other ones; thus, the buyer is not going to own bonds whose return, for a given period of time, will be lower in comparison with other bonds that are perfect substitutes. In fact, if bonds are perfect substitutes, their interest rate is going to be rigorously equal.

To understand why these hypotheses - referring to perfect substitutes between bonds that have different maturities - lead to the anticipation theory, we suggest considering two investment strategies, i.e.:

1) Buying a 1 -year bond, and buying a newly issued one, for a 1 -year term; in other words, these bonds have a 1 -year maturity.
2) Buying a 2 -year bond and owning it till it reaches maturity.

Because there are two bonds that are owned, they will have the same anticipated return and the interest rate for 2 -year bonds will be equal with the average of the two 1 -year bonds. If, for example, the interest rate for 1 year is $9 \%$ and if we foresee that the 1 -year interest rate will be $11 \%$, the first strategy leads to an anticipated 2 -year return $(9 \%+11 \%) / 2$ of $10 \%$ per year.

A buyer is not indifferent to the two strategies because the interest rate for 2 -year bonds is $10 \%$. The substitute nature of the two strategies reveals that the interest rate for 2 years is the average of two interest rates that are owned for 1 year successively.

Generally speaking, for a 1 Euro placement, one chooses, for two periods, either to own a bond whose maturity is a two-period term or successively two bonds which have the maturity for each period. The variables are the following ones:
$-i_{t}=$ the interest rate today (time $t$ ) for a bond whose maturity is established for one period.
$-i_{t+1}^{e}=$ the anticipated interest rate today for a close maturity (time $t+1$ ) for a bond whose maturity is established for one period.
$-i_{2 t}=$ today's interest rate of a bond with a two-period maturity.
The anticipated return for the two periods of a 1-Euro placement for bonds that are owned during two periods of time, and preserved during the 2 -period term, is equal with:
$\left(1+i_{2 t}\right)\left(1+i_{2 t}\right)-1=1+2 i_{2 t}+\left(i_{2 t}\right)^{2}-1=2 i_{2 t}+\left(i_{2 t}\right)^{2}$
Consequently, the value of the 1-Euro placement for a 2 -period term is equal with:

$$
\left(1+i_{2 t}\right)\left(1+i_{2 t}\right)
$$

The return is equal with the diminished volume of the 1-Euro placement in relation to the initial placement, which is of 1 Euro.

Since " $i_{2 t}^{e}$ " it is, in general, very low in relation to $2 i_{2 t}$, it can simplify and consider that the anticipated return for two periods is of $2 \mathrm{i}_{2 \mathrm{t}}$.

If we adopt another investment strategy, successively buying two bonds with a maturity for a certain period, the anticipated return for a 1-Euro investment that have a two-period term will be:

$$
\left(1+i_{t}\right)\left(1+i_{t+1}^{e}\right)-1=1+i_{t}+i_{t+1}^{e}+i_{t}\left(i_{t+1}^{e}\right)-1=i_{t}+i_{t+1}^{e}+i_{t}\left(i_{t+1}^{e}\right)
$$

Thus, after a certain period, the 1-Euro investment becomes " $1+i_{t}$ " Euro, which, reinvested in a new bond becomes" $i_{t+1}^{e}$ " what we give $(1+i)\left(1+i_{t+1}^{e}\right)$. Deducing the Euro of the initial placement and dividing through the volume (still 1 Euro) of the initial placement, we obtain the anticipated return of this strategy. Since for a moderate interest rate, the outcome " $i_{t}$ " through " $i_{t+1}^{e}$ " is low (if $i_{t}=i_{t+1}^{e}=10 \%$, therefore 0,01 ), one can simplify and consider that the anticipated return for two period amounts at:

$$
i_{t}=i_{t+1}^{e}
$$

If we could resort to an arbitrage, there would not be holders for all bonds considering that the return for these two placement strategies were similar; in other words:

$$
2 i_{2 t}=i_{t}+i_{t+1}^{e}
$$

Hence:

$$
i_{2 t}=\frac{\left(i_{t}+i_{t+1}^{e}\right)}{2}
$$

Thus, the rate for bonds whose two-period maturity is equal with the average for the two rates for one period.

The same return could be obtained for bonds whose maturity is more important and for an important number of periods, as wished. Thus, we find that the interest rate marked with " i " of a bond with a maturity for " n " periods that may be equal are written as follows:

$$
i_{n t}=\frac{i_{t}+i_{t+1}^{e}+i_{t+2}^{e}+. .+i_{t(n-1)}^{e}}{n}
$$

In other words, the interest rate of a bond with " $n$ " periods will be equal with the average of the interest rate of a period, as well as with the interest rate of a bond for an anticipated period, respectively for " n " successive periods. The anticipation theory for the rate curve is precisely expressed in this manner.

An example could facilitate understanding. Let us suppose that the interest rates for an anticipated year for every 5 successive years are subsequently: $5 \%, 6 \%, 7 \%, 8 \%$ and $9 \%$, through equation 2 ; the interest rate for a 2 -year bond could be:
$\frac{5 \%+6 \%}{2}=5,5 \%$
The interest rate for a 5 -year bond could be:

$$
\frac{5 \%+6 \%+7 \%+8 \%+9 \%}{5}=7 \%
$$

The anticipation theory is an elegant theory that explains why the rate curve can have various shapes. It suggests that if the interest rate curve is ascending, this is due to the fact that it anticipates an increase of the short-term interest rate for the future. Vice versa, when the interest rate curve is descending, the anticipation theory suggests that the short-term interest rate will be descending in the future.

It does not foresee stability for the interest rate if the rate curve is flat.
The anticipation theory explains for the first time the facts that we previously presented and it refers to the interest rate of bonds that have different maturities which interchange in time in parallel. Historically, the interest rate has the following characteristics: if it is ascending today, it tends to ascend in the future; consequently, if it is descending today, it tends to ascend the anticipated interest rate in the future.

The long-term interest rate is the future average of short-term rates; an increase of the short-term interest rate influences the long-term interest rate, which leads to parallel variations.

The anticipation theory explains the second fact for it is known that the rate curves tend to be ascending when the interest rate is low and descending when the interest rates are high.

Consequently, when rates are low, one generally anticipates that they will be higher in the future, and that they will later have an average or "normal" level; similarly, the anticipated interest rates for the future are higher to the short-term interest rates which are currently very high; one can anticipate that the interest rate will be lower and will be again "normal" in the future. This makes today's long-term rate to be even lower in comparison with the short-term interest rate, which is illustrated as an inverted yield curve.

The anticipation theory must also simply explain a large number of characteristics of the structure through the maturity of interest rates. Unfortunately, it does not explain the third fact presented before: the interest rate follows a normally ascending trend.

Consequently, according to the anticipation theory, for a normal anticipation it is recommended to use a high level of the short-term interest rate.

The anticipation theory supposes that there are systematical errors of anticipations, which is quite unacceptable. In order to be keep up with the historical evolution of the interest rate, the rate curves become, according to this theory, flat averages, i.e. curves which do not ascend.

### 3.2. Segmented markets theory

According to the theory of segmented markets, the markets of bonds that have varied maturities are segmented or evolve separately. The price for each of these markets and the interest rate for bonds that have their own maturity is determined in relation to supply and demand, without having an effect on the anticipated returns of bonds that have different maturities.

The key hypothesis of this theory is that bonds with differential maturity cannot be substituted, which leads to the fact that the anticipated return of a bond with a certain maturity does not have any effect on the demand for a bond with a different maturity. This hypothesis is quite different from the anticipation theory, according to which bonds with varied maturities can be perfectly interchanged.

The argument that does not support the substitution of bonds which have different maturities is that investors have important arguments when they choose a certain maturity or another one. This happens because they think of a precise placement duration and they wish that their placement maturity is identical for eliminating risk (when the maturity is equal with the placement duration, the return is well-known and it is equal with the actuarial interest rate; thus, the interest rate risk is reduced to zero).

The theory of segmented markets explains the differentials of rate curve shapes through the different supply and demand trends between the different maturities bonds markets. It is likely for investors to have in general a preference for shorter placement duration; we refer to loan duration; thus, the balance interest rate on the bonds market for short-term bonds is lower than the one for long-term bonds.

Similarly, the theory of the segmented markets explains the third statistical fact that we referred to above, for it considers as it is well-known that short-term interest rates are in general lower in comparison with long-term interest rates; in other words, the rate curve is in general ascending.

The theory of segmented markets explains the ascending rate curves but it does not explain the first two facts. In consequence, if markets for bonds with different maturities are previously separated, this is not sensible because interest rates for bonds with different maturities vary in general. The relation between different maturities and the rate level does explain why the interest rate curve tends to be ascending when rates are low and descending (or vice versa) when rates are high.

Since each of the two theories referring to the structure through the maturity of interest rates partially explains facts, it is logical for them to be combined with the liquidity premium theory so that together they could offer a general explanation.

### 3.3. The liquidity premium theory and preferred areas theory

### 3.3.1. The liquidity premium theory

According to the liquidity premium theory, the interest rate for a long-term bond is equal with the average of anticipated short-term interest rates calculated for the whole duration of a bond, whereas the increase of a liquidity premium depends on the supply and offer that exist on the market for these bonds.

The key hypothesis for the liquidity premium theory is that bonds with different maturities can be substituted, a fact which proves that the anticipated return of a certain type of bonds influences the price of another type of bonds; however, this substitution is irreversible because investors can prefer bonds with a certain maturity to bonds that have a different maturity. Similarly, investors prefer short-term bonds because they pose a lower risk for the interest rate.

Actually, long-term bonds are not owned by investors because their return has a positive liquidity premium, which compensates their shortcomings in relation to short-term bonds. Similarly, the anticipation theory is modified with the help of a liquidity premium in the equation and in relation to long-term interest rates and the anticipated short-term interest rates; the equation if written as follows:

$$
I_{n t}=\frac{i_{T}+i_{t+1}^{e}+i_{t+2}^{e}+. .+i_{t(n-1)}^{e}}{n}+i_{n t}
$$

where:
$I_{n t}=$ the liquidity premium of a bond with a maturity for " n " periods and for the moment " t ".
This theory, i.e. the liquidity premium theory, corrects a lack in the previous theory, i.e. the fact of knowing the indifference attributed to investors in relation to the maturity of the bonds that they own.
According to the theory created by J. Hicks in 1939, the long-term rate takes into account the future short-term interest rates anticipations and also contain a liquidity premium:

$$
o r_{T}=\sqrt[T]{\left(1+o r_{1}\right)\left(1+E\left(1 r_{1}\right)+1 L_{1}\right)\left(1+E\left(2 r_{1}\right)+2 L_{1}\right) \ldots+\left(1+E\left(1 r_{1}+T-1 L_{1}\right)\right)-1}
$$

where:
$o r_{T}=$ the noticeable actual interest rate
$1 L_{1}, 2 L_{1}=$ the liquidity premium which investors have for accepting the risk posed by the prolongation of maturity for the bonds that they hold for each supplemented year.

This is so because the volatility of the bonds price increases with its maturity in a descending rhythm with ${ }_{1} \mathrm{~L}_{1}<{ }_{\mathrm{T}-1} \mathrm{~L}_{1}<_{\mathrm{T}-2} \mathrm{~L}_{2}, \ldots$

The different values for " L " remain positive, however.
Liquidity premiums make the structure of the rate have the same framework if the future short-term rates are stable; the ascending trend of this structure is stronger if there is a decrease of the rate and the decreasing curve of the structure is stronger if there is a decrease of the rate, whereas the ascending curve of the structure is diminished and it will even follow an ascending trend.

### 3.3.2. Preferred areas theory

This theory is quite close to the liquidity premium theory. It also modifies the anticipation theory. It supposes that investors prefer bonds with a certain maturity, which respond to a preferred localization. The reason why they prefer bonds with a certain maturity is represented by the fact that they do not accept to hold bonds with different maturities whose return is higher.

Since, in general, investors prefer short-term maturities to the long-term ones, they do not accept to own long-term bonds whose anticipated returns are higher and lead to the same equation as it happens with the liquidity, i.e. normally ascending together with the maturity.

The relation between the anticipation theory and the liquidity premium theory (or the preferred areas theory) is presented in picture 2 . Since the liquidity premium is positive, it (normally) increases with maturity, whereas the rate curve foreseen by the liquidity premium is however low in comparison with the one foreseen by the anticipation theory and it is, in general, ascending on the curve that evolves faster.

Figure 2. The relation between liquidity and the theory of anticipation


Liquidity premium theory (or the theory of preferred areas) facilitates the explanation of the three formerly presented facts. Thus, it explains that the interest rates of bonds that have different maturity terms vary in parallel in the course of time because an increase of the short-term interest rate illustrates that this rate is going to be higher on the average in the future, a fact which implies an increase of the long-term interest rate, as well.
It also explains that the interest rate curve tends to be faster when the rate is very low and vice versa when the rate is very high. This fact generally makes investors anticipate that the short-term rate will increase when the average of the future short-term interest rates is abnormally low, thus becoming higher to the present ones. With the increase of a liquidity premium, the long-term interest rates are going to be significantly low. Vice versa, if short-term interest rates are very high, the anticipation of their reduction can be thus through the liquidity premium, so that long-term rates are lower than short-term rates, generating an inverted yield curve.

Premium liquidity theory facilitates market anticipations for the future short-term interest rates, which are partially noticeable with the yield curve. An ascending curve indicates that the short-term interest rate is going to go up in the future. A moderately ascending curve indicates that the rate is going to be relatively stable; a flat curve points out that short-term interest rates are going to moderately go down. Finally, an inverted yield curve indicates that the interest rate is going to be forced to go down.

## 4. Conclusions

The structure through the interest rate risk, in other words the relationship between the interest rate for bonds which have the same maturity is explained by three factors: nonpayment risk, liquidity and fiscality. When the nonpayment risk of a bond is increased, the risk premium (the difference between the interest rate and the riskless payment Treasury bonds) increases, too. The higher liquidity of Treasury bonds is also justified by the fact that their interest rate is lower than the one of securities with lower liquidity. Finally, a favorable fiscal treatment, as the one of American municipal bonds, lead to a lower interest rate.

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