National and International Inequalities in Income and Wealth in a Global Growth with Free Trade and National Inflation Policies

WEI-BIN ZHANG
Professor, Associate Dean of International Cooperation and Research Division
Ritsumeikan Asia Pacific University, Yokohama
JAPAN
wbz1@apu.ac.jp

Abstract

The purpose of this paper is to study global monetary economic growth with heterogeneous households under free trade. The paper examines dynamics of global and national wealth and income distribution in association with monetary economic growth within an integrated framework. Money is introduced via the cash-in-advance (CIA) approach. We show that the dynamics of the world economy (with any number of countries) is described by a set of differential equations. We simulate equilibrium of the global economy with three countries and two types of households in each country. We also demonstrate effects of changes in technology and inflation policy. Our model demonstrates, as Grier and Grier (2007) empirically show, that the global economy exhibits absolute divergence in output levels if some determinants of steady state income are different. The study shows that as one country increases its inflation policy, the equilibrium values of the global output, consumption level and physical wealth are enhanced, and the rate of interest is lowered. The country which raises its inflation policy benefits in every aspect, but the other countries suffer in some aspects and benefit in others.

Keywords: *money; global growth; trade pattern; global and national distribution of income and wealth.*

JEL Classification: F11; O42.

1. Introduction

This study examines dynamic interactions among economic growth, inflation policies and international trade in a heterogeneous household framework. Because of rapidly increasing complexity of financial markets in association with globalization and wide spread of computer in recent years, financial markets have increasingly become complicated. In order to properly address issues related to global economic growth, it is important to study growth and money in an integrated framework. Nevertheless, many of economic dynamic models in international economics omit monetary issues, by explicitly or implicitly assuming that transactions on the economy's real side can be carried out frictionlessly without money. On the other hand, it is well known that there are many studies on interactions among growth and money in macroeconomics. Modern analysis of the long-term interaction of inflation and capital formation begins with Tobin's seminal contribution (Tobin, 1956). Tobin (1965) deals with an isolated economy in which "outside money" competes with real capital in the portfolios of agents by extending the Solow growth model. Tobin (1965: 676) argues: "The community's wealth … has two components: the real goods accumulated through past real investment and fiduciary or paper 'goods' manufactured by the government from thin air. Of course the non-human wealth of such a nation 'really' consists only of its tangible capital. But, as viewed by the inhabitants of the nation individually, wealth exceeds the tangible capital stock by the size of what we might term the fiduciary issue. This is an illusion, but only one of the many fallacies of composition which are basic to any economy or any society. The illusion can be maintained unimpaired so long as the society does not actually try to convert all of its paper wealth into goods." Tobin's model includes a real sector as in the Solow growth model. In the monetary economy prices are expressed in money, transactions require money, and financial wealth can be held in the form of money or financial instruments competing with money. In the Tobin model, money is a liability of the public sector. As a depositor of purchasing power money can be held by private agents as an alternative form of wealth to physical capital stock. Different from a barter economy as described by the Solow model, the Tobin model involves a problem of deciding the optimal composition of wealth at every instant. Since Tobin published his model, many other monetary growth models for national economies have been built. For instance, Sidrauski (1967) constructed an economic model in which no real variable will be affected by the economy's inflation rate. We will address the issues by Tobin and Sidrauski in the alternative framework. Our approach is based on the cash-in-advance (CIA) approach. Clower (1967) proposed a model to incorporate the role of money as a medium of exchange through the CIA constraint. The basic idea is to explain the role that money plays in carrying out transactions by introducing transaction technology. Stockman (1981) proposes another growth model through CIA constraints. The model predicts that there is long-run superneutrality if only consumption expenditures are subject to a CIA constraint. If investment is also subject to a CIA constraint then steady state capital will fall when the growth rate of money rises. Marquis and Reffett (1991) and Mino and Shibata (1995) also introduce money into two-sector models involving human capital via a cash-in-advance constraint. It has become clear that different approaches of taking account of money in growth models lead to incompatible effects of inflation on capital accumulation and wealth and income distribution. There are many other studies which apply cash-in-advances (for instance, Lucas and Stokey, 1987; Townsend, 1987; Woodford, 1994; Santos, 2006; Chen et al., 2008; Miyazaki, 2012; Kam, 2013; Chang et al. 2013). Irrespective of these efforts, only a few models are proposed to study effects of monetary policies on global growth and international trade.

Our main interest is to show how monetary policies affect global growth and trade patterns. Trade among countries has been increasingly expanded both in volume and variety. Some empirical studies affirmatively support positive impact of trade on global economic growth. For instance, a study by Chang et al. (2009) demonstrates that the positive effects of trade openness may be greatly improved under certain conditions. Other empirical studies show the opposite. For instance, Yanikkaya (2003) empirically demonstrates that trade liberalization does not have a simple and straightforward relationship with economic growth. Contrary to the conventional view on the growth effects of free trade, the estimation results show that trade liberalization may not be positively related to economic growth, especially for developing economies. There are many empirical studies on relations between trade and growth (see, for instance, Edwards, 1993; Sachs and Warner, 1995; Krueger, 1998; Shilimbergo, et al. 1999; Rodriguez and Rodrik, 2001; Yanikkaya, 2003; Lee at al. 2004; Chang et al. 2009; Antonakakis, 2012; Obrizan 2013). It is evident that in order to properly analyze these important issues related to trade, growth and distribution, we need a dynamic model of growth and trade with income and wealth distributions within and among countries. This paper attempts to develop an international monetary growth model with capital accumulation and heterogeneous households in each country. As far as growth and trade are concerned, this study is based on the traditional dynamic one-commodity and multiple-country growth trade with perfect capital mobility. It is well known that since the publication of the Oniki-Uzawa model of trade and economic growth by Oniki and Uzawa (1965), various trade models with endogenous capital have been proposed (for instance, Deardorff, 1973; Ruffin, 1979; Findlay, 1984; Frenkel and Razin, 1987;

Eaton, 1987; Frankel and Romer, 1999; Brecher, et al. 2002; Nishimura and Shimomra, 2002; Sorger, 2002; Farmer and Lahiri, 2005; Doi et al. 2007; Lee, 2011; Zhang, 2013). Nevertheless, almost all of these studies are concerned with two-country cases without money. There is a need to generalize the model to multiple countries. As reviewed by Lee (2011: 260), "Innumerable articles and volumes have been published to extend the Ramsey-type endogenous growth model to various directions. … However, only a few contributions extend these models to a two-country or multi-country economy to re-examine the trade issues and the long-run growth rate jointly in a unified framework." This study introduces money into a multi-country heterogeneous-household growth model with free trade and perfect competition. The paper is a synthesis of a multi-national growth model by Zhang (1994) and the monetary growth model of a national economy with the CIA approach by Zhang (2009: Chap. 4). The paper is organized as follows. Section 2 defines the multi-country monetary growth model with capital accumulation and free trade. Section 3 shows that the dynamics of the world economy with any number of countries can be described by a set of differential equations. Section 4 simulates the equilibrium of a world economy with 3 countries and 2 types of households in each country. Section 5 examines the effects of changes in some parameters. Section 6 concludes the study.

2. The Multi-Country Trade Model with Money and Capital Accumulation

In describing economic production, we follow the neoclassical trade framework. Most aspects of production sectors in our model are similar to the neoclassical one-sector growth model (for instance, Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). It is assumed that the countries produce a homogenous commodity (e.g., Ikeda and Ono, 1992). There is only one (durable) good in the global economy under consideration. Production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. A national economy has two assets, domestic money and traded capital goods. The economy consists of consumers, firms and the government. The foreign price of traded goods is given in the world market. The domestic residents may hold two assets, domestic money and traded goods (or world bond). We neglect transport cost, customs, or any other possible impediments to trade. We have perfect mobility of goods. The system consists of multiple countries, indexed by $j = 1, ..., J$. We assume that there is no migration between the countries and the labor markets are perfectly competitive within each country. Each country has a fixed labor force, $\bar{N}_j$ ($j = 1, ..., J$). We further classify each country's population into Q_j groups. We assume that each group has a fixed population, $\bar{N}_{jq}$, $j = 1, ..., J$, $q = 1, ..., Q_{q_j}$. We have

$$\bar{N}_j = \sum_{q=1}^{Q_j} \bar{N}_{jq} .$$

We use h_{jq} to represent for the level of human capital of household (j, q). In this study we assume human capital exogenous. The total labor supply of country j is

$$N_j = \sum_{q=1}^{Q_j} h_{jq} \bar{N}_{jq} , \quad j = 1, ..., J .$$

We denote wage rate of group (j, q) and interest rates by $w_{jq}(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$.

Behavior of firms

First, we describe behavior of the production sections. We assume that there are only two productive factors, capital $K_j(t)$ and the total labor force, N_j. The production functions are given by $F_j(K_j(t), N_j)$, $j = 1, \cdots, J$, where F_j are the output of country j. Assume F_j to be neoclassical. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest $r(t)$ and wage rates $w_{jq}(t)$ are determined by markets. The marginal conditions are given by

$$r + \delta_{kj} = f_j^{'}(k_j), \ \ w_{jq}(t) = h_{jq}\, w_j(t), \ \ w_j(t) = f_j(k_j) - k_j f_j^{'}(k_j), \tag{1}$$

where δ_{kj} are the depreciation rate of physical capital in country j, and $k_j \equiv K_j / N_j$ and $f_j(k_j) \equiv F_j(k_j, 1)$.

Introduction of money

We assume that each country's money is a nonetradeable asset. This is a strict requirement for modern economies. We make this assumption for simplicity of analysis. In each national economy the scheme according to which the money stock evolves over time is deterministic and known to all agents. We first assume that a central bank of country j distributes at no cost to the population a per capita amount of fiat money $M_j(t) > 0$. With μ_j being the constant net growth rate of the money stock, $M_j(t)$ evolves over time according

$$\dot{M}_j(t) = \mu_j M_j(t), \ \ \mu_j > 0.$$

The government brings $\mu_j M_j(t)$ additional units of money per capita into circulation in order to finance all government expenditures via seigniorage. Let $m_j(t)$ stand for the real value of money per capita measured in units of the output good, that is, $m_j(t) = M_j(t)/P_j(t)$. The government expenditure in real terms per capita, $\tau_j(t)$, is $\tau_j(t) = \mu_j m_j(t)$. The representative household of each group receives $\mu_j m_j(t)$ units of paper money from the government through a "helicopter drop", also considered to be independent of his money holdings.

Behavior of consumers

This study applies an alternative approach to household proposed by Zhang (1993). Applications of this approach to different economic problems are extensively discussed by Zhang (2009). We refer to Zhang's book for further explaining constrain and utility function. Consumers make decisions on consumption levels of services and commodities as well as on how much to save. Let $\pi_j(t)$ and $\bar{k}_{jq}(t)$

respectively stand for the inflation rate and the per capita wealth of household q in country j. The current income of household q in country j is given by

$$y_{jq}(t) = r(t)\bar{k}_{jq}(t) + w_{jq}(t) - \pi_j(t)m_{jq}(t) + \mu_j m_j(t), \quad j = 1, \ldots J, \quad q = 1, \ldots, Q_{q_j}, \tag{2}$$

where $r(t)\bar{k}_{jq}(t)$ is the interest payment, $\pi_j(t)m_{jq}(t)$ is the real cost of holding money, and $\mu_j(t)m_j(t)$ is the real value of paper money from the government. In Zhang's approach, th disposable income is the current income plus the value of wealth held by the household. The value of wealth held by the household is denoted by $a_{jq}(t) = \bar{k}_{jq}(t) + m_{jq}(t)$. The disposable income is given by

$$\hat{y}_{jq}(t) = y_{jq}(t) + a_{jq}(t),$$

When deciding the composition of their portfolios, the household knows in advance that a certain fraction of consumption needs to be financed by payment in cash. Assume that cash has to be held in advance of purchasing goods. The liquidity constraint of the household is formed as

$$m_{jq}(t) = \chi_{jq} c_{jq}(t),$$

where χ_{jq} are positive parameters. We require $0 < \chi_{jq} < 1$. Substituting this equation into $\hat{y}_{jq}(t) = y_{jq}(t) + a_{jq}(t)$, we have

$$\hat{y}_{jq}(t) = (1 + r(t))\bar{k}_{jq}(t) + w_{jq}(t) + (1 - \pi_j(t))\chi_{jq} c_{jq}(t) + \mu_j m_j(t). \tag{3}$$

At each point of time, a consumer distributes the total available budget between saving, $s_{jq}(t)$, consumption of goods, $c_{jq}(t)$. The budget constraint is given by

$$c_{jq}(t) + s_{jq}(t) = \hat{y}_{jq}(t).$$

From this equation and equation (3), we have

$$(\bar{\chi}_{jq} + \chi_{jq}\pi_j(t))c_{jq}(t) + s_{jq}(t) = \bar{y}_{jq}(t) \equiv (1 + r(t))\bar{k}_{jq}(t) + w_{jq}(t) + \mu_j m_j(t), \tag{4}$$

where $\bar{\chi}_{jq} \equiv 1 - \chi_{jq}$. The utility level of household q in country j is represented by

$$U_{jq}(t) = \theta_{jq} c_{jq}^{\xi_{0jq}}(t) s_{jq}^{\lambda_{0jq}}(t), \quad \xi_{0jq}, \lambda_{0jq} > 0,$$

in which ξ_{0jq}, and λ_{0jq} are a person's elasticity of utility with regard to commodity and savings in country j. We call ξ_{0jq} and λ_{0jq} propensities to consume goods and to hold wealth (save), respectively. Maximizing $U_{jq}(t)$ subject to (4) yield

$$(\bar{\chi}_{jq} + \chi_{jq}\pi_j(t))c_{jq}(t) = \xi_{jq}\bar{y}_{jq}(t), \quad s_{jq}(t) = \lambda_{jq}\bar{y}_{jq}(t), \tag{5}$$

where $\xi_{jq} \equiv \rho_{jq}\,\xi_{0jq}$, $\lambda_{jq} \equiv \rho_{jq}\,\lambda_{0jq}$, $\rho_{jq} \equiv \dfrac{1}{\xi_{0jq} + \lambda_{0jq}}$.

According to the definitions of $s_{jq}(t)$, the wealth accumulation of the representative person in country j is given by

$$\dot{a}_{jq}(t) = s_{jq}(t) - a_{jq}(t). \tag{6}$$

These equations simply imply that the change in a household' wealth is the saving minus dissaving (e.g., Zhang, 2009).

Inflations and changes in money

According to the definitions of $\pi_j(t)$ and $m_j(t)$, we have

$$\dot{m}_j(t) = (\mu_j - \pi_j(t))m_j(t). \tag{7}$$

The global wealth being fully employed

The total capital stock employed by the production sectors is equal to the total wealth owned by all the countries. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J}\sum_{q=1}^{Q_j} \bar{k}_{jq}(t)\bar{N}_{jq}. \tag{8}$$

Money demand and supply

The total demand for money is equal to the total supply in each country

$$\sum_{q=1}^{Q_j} m_{jq}(t)\bar{N}_{jq} = \bar{N}_j m_j(t). \tag{9}$$

Trade balances

We now describe trade balances of the countries. If $\bar{K}_j(t) - K_j(t) > (<)\ 0$, we say that country j is in trade surplus (trade deficit). If $\bar{K}_j(t) - K_j(t) = 0$, we see that country j is in trade balance. We introduce variables to measure trade balances

$$E_j(t) \equiv r(t)\left(\bar{K}_j(t) - K_j(t)\right).$$

We have thus built the model which explains the endogenous accumulation of capital and the international distribution of capital in the world economy in which the domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile and labor is internationally immobile. We now examine the properties of the system.

3. Dynamics and Equilibrium of the Global Economy

This section shows that the dynamics of the world economy can be expressed as a set of differential equations. The following lemma is proved in the appendix.

Lemma 1

Let $\chi_{jq} = \chi_j$ for all q. The dynamics of the world economy is given by the following differential equations $m_j(t)$, $k_1(t)$ and $\bar{k}_{jq}(t)$, $(j, q) \neq (1, 1)$, as the variables

$$\dot{m}_j(t) = \Psi_j\left(k_1(t), \{\bar{k}(t)\}, m_j(t)\right),$$

$$\dot{k}_1(t) = \Psi_k\left(k_1(t), \{\bar{k}(t)\}, [m_j(t)]\right),$$

$$\dot{\bar{k}}_{jq}(t) = \hat{\Psi}_{jq}\left(k_1(t), \{\bar{k}(t)\}, [m_j(t)]\right), \quad j = 1, ..., J, \quad q = 1, ..., Q_j, \quad (j, q) \neq (1, 1), \tag{10}$$

in which functions Ψ_j, Ψ_k, and $\hat{\Psi}_{jq}$ are defined in the appendix. For any given positive values of $m_j(t)$, $k_1(t)$ and $\bar{k}_{jq}(t)$ at any point of time, the other variables are uniquely determined by the following procedure: $\pi_j(t)$ by (A5) $\to$ $m_{jq}(t)$ by (A6) $\to \bar{k}_{11}(t)$ by (A2) $\to$ $a_{jq}(t) = \bar{k}_{jq}(t) + m_{jq}(t)$ $\to k_j(t) = \phi_j(k_1(t))$ $\to$ $f_j(t) = f_j(k_j(t))$ $\to$ $r(t)$ and $w_{jq}(t)$ by (1) $\to$ $\bar{y}_{jq}(t)$ by (A3) $\to$ $c_{jq}(t)$ and $s_{jq}(t)$ by (5) $\to$ $F_j(t) = N_j(t)\, f_j(t)$.

This lemma is important as it gives a procedure for the computer to simulate the motion of the global economy. Although we may analyze behavior of the high dimensional differential equations, it is difficult to explicitly interpret results. For illustration, we specify the production functions as follows:

$$F_j(t) = A_j K_j^{\alpha_j}(t) N_j^{\beta_j}, \ \alpha_j + \beta_j = 1, \ \alpha_j, \beta_j > 0,$$

where A_j is country j 's productivity and α_j is a positive parameter. From equations $k_j(t) = \phi_j(k_1(t))$ and $f_j(t) = A_j\, k_j^{\alpha_j}(t)$, we have

$$\phi_j(k_1(t)) = \left(\frac{\alpha_1 A_1 k_1^{-\beta_1}(t) - \delta_j}{\alpha_j A_j}\right)^{-1/\beta_j}, \quad \bar{\phi}_j(k_1(t)) = A_j\, \beta_j\, \phi_j^{\alpha_j}(k_1(t)), \quad j = 1, \cdots, J. \tag{11}$$

We show how to determine equilibrium of the dynamic system. First by (7), we have $\pi_j = \mu_j$ at equilibrium. By (6), we have $s_{jq} = \bar{k}_{jq} + m_{jq}$. From $s_{jq} = \lambda_{jq}\bar{y}_{jq}$, $s_{jq} = \bar{k}_{jq} + m_{jq}$ and the definition of $\bar{y}_{jq}$, we obtain

$$m_{jq} = \left(\lambda_{jq} + \lambda_{jq}\, r - 1\right)\bar{k}_{jq} + \lambda_{jq}\, w_{jq} + \lambda_{jq}\, \mu_j\, m_j. \tag{12}$$

Multiplying the two sides of (12) by $\bar{N}_{jq}$ and then adding the resulted Q_j equations for each j, we have

$$\sum_{q=1}^{Q_j} \bar{N}_{jq} m_{jq} = \sum_{q=1}^{Q_j} \left[\left(\lambda_{jq} + \lambda_{jq} r - 1 \right) \bar{k}_{jq} + \lambda_{jq} w_{jq} \right] \bar{N}_{jq} + \left(\mu_j \sum_{q=1}^{Q_j} \lambda_{jq} \bar{N}_{jq} \right) m_j . \tag{13}$$

From (13) and (9), we solve

$$m_j \left(k_1, \{\bar{k}\} \right) = \sum_{q=1}^{Q_j} \tilde{r}_{jq} \bar{k}_{jq} + \hat{N}_j , \tag{14}$$

where

$$\tilde{r}_{jq} \equiv \tilde{n}_j r_{jq} \bar{N}_{jq}, \ \ \hat{N}_j \equiv \tilde{n}_j \sum_{q=1}^{Q_j} \bar{N}_{jq} \lambda_{jq} w_{jq}, \ \ r_{jq} \equiv \lambda_{jq} + \lambda_{jq} r - 1, \ \ \tilde{n}_j \equiv \frac{1}{\bar{N}_j - \left(\mu_j \sum_{q=1}^{Q_j} \lambda_{jq} \bar{N}_{jq} \right)}.$$

According to (A3) and (14), we see that we can explicitly express $\bar{y}_{jq}$ as functions of k_1 and $\{\bar{k}\}$. In studying equilibrium, we don't make the assumption that all the households within a country have the equal rate of χ_{jq}. From the definition of $\bar{y}_{jq}$, $m_{jq} = \chi_{jq} c_{jq}$, $\left(\bar{\chi}_{jq} + \chi_{jq} \pi_j \right) c_{jq} = \xi_{jq} \bar{y}_{jq}$, and $\pi_j = \mu_j$, we solve

$$m_{jq} = W_{jq} \bar{k}_{jq} + \bar{\xi}_{jq} w_{jq} + \mu_j \bar{\xi}_{jq} m_j , \tag{15}$$

where $W_{jq} \equiv (1 + r) \bar{\xi}_{jq}$, $\bar{\xi}_{jq} \equiv \dfrac{\chi_{jq} \xi_{jq}}{\bar{\chi}_{jq} + \chi_{jq} \mu_j}$.

According to (A3) and (14), we see that we can explicitly express m_{jq} as functions of k_1 and $\{\bar{k}\}$. From (15) and $\left(\bar{k}_{jq} + m_{jq} \right) / \lambda_{jq} = (1 + r) \bar{k}_{jq} + w_{jq} + \mu_j m_j$ (which is from $s_{jq} = a_{jq}$), we solve

$$\bar{k}_{jq} = \bar{W}_{jq} + R_{jq} m_j , \tag{16}$$

where we use (15) and

$$\bar{W}_{jq} \equiv \frac{\bar{\mu}_{jq} w_{jq}}{1 + W_{jq} - (1 + r) \lambda_{jq}}, \ \ R_{jq} \equiv \frac{\bar{\mu}_{jq} \mu_j}{1 + W_{jq} - (1 + r) \lambda_{jq}}, \ \ \bar{\mu}_{jq} \equiv \lambda_{jq} - \bar{\xi}_{jq} .$$

We also note that w_{jq} and r are functions of k_1. From (14) and (16), we have

$$\bar{k}_{jq} = \bar{W}_{jq} + R_{jq} \hat{N}_j + R_{jq} \sum_{q=1}^{Q_j} \tilde{r}_{jq} \bar{k}_{jq} . \tag{17}$$

The equations are linear in $\bar{k}_{jq}$. It can be seen that for each j, we have Q_j linear equations containing Q_j variables, $\bar{k}_{j1}, \ldots, \bar{k}_{jQ_j}$. Assume that from (17) we can solve $\bar{k}_{jq}$ as functions of k_1, denoted by, $\bar{k}_{jq} = \Omega_{jq}(k_1)$. Inserting $\bar{k}_{jq} = \Omega_{jq}(k_1)$ and $k_j = \phi_j(k_1)$ in (8), we have

$$\Omega(k_1) \equiv \sum_{j=1}^{J} \phi_j(k_1) N_j - \sum_{j=1}^{J} \sum_{q=1}^{Q_j} \Omega_{jq}(k_1) \bar{N}_{jq} = 0. \tag{18}$$

Lemma 2

We determine equilibrium of the dynamic system by the following procedure: $\pi_j = \mu_j \rightarrow k_1$ by (18) $\rightarrow$ k_j and w_{jq} by (A1) $\rightarrow$ $\bar{k}_{jq}$ by (17) $\rightarrow$ m_j by (14) $\rightarrow$ m_{jq} by (15) $\rightarrow$ $a_{jq} = \bar{k}_{jq} + m_{jq}$ $\rightarrow$ $f_j = f_j(k_j) \rightarrow r$ by (1) $\rightarrow$ $\bar{y}_{jq}$ by (A3) $\rightarrow$ c_{jq} and s_{jq} by (5) $\rightarrow$ $F_j = N_j f_j$.

As it is difficult to examine dynamic behavior of the high dimensional dynamic system, we are only concerned with steady states in the rest of the paper.

4. Equilibrium with Three Countries and Two Groups in Each Country

For illustration, we will follow the procedure given in Lemma 1 to examine equilibrium of the global economic system. For simulation, we specify the production functions $F_j = A_j K_j^{\alpha_j} N_j^{\beta_j}$. We specify

$$\chi_{jq} = 0.5, \ \delta_{kj} = 0.05, \ \lambda_{jq} + \xi_{jq} = 1,$$

and the other parameters as follows

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}, \ \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.04 \\ 0.05 \end{pmatrix}, \ \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 0.3 \\ 1/3 \end{pmatrix},$$

$$\begin{pmatrix} \lambda_{11} \\ \lambda_{12} \\ \lambda_{21} \\ \lambda_{22} \\ \lambda_{31} \\ \lambda_{32} \end{pmatrix} = \begin{pmatrix} 0.85 \\ 0.8 \\ 0.77 \\ 0.74 \\ 0.75 \\ 0.72 \end{pmatrix}, \ \begin{pmatrix} N_{11} \\ N_{12} \\ N_{21} \\ N_{22} \\ N_{31} \\ N_{32} \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 5 \\ 8 \end{pmatrix}, \ \begin{pmatrix} h_{11} \\ h_{12} \\ h_{21} \\ h_{22} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \\ 1 \\ 0.6 \end{pmatrix}. \tag{19}$$

Group 1 in Country has the highest level of human capital and highest propensity to save. Country 1's population is less than that of country 2. The human capital level of group 1 in country 2 is the second, next to country 1's. Country 3 has the largest population and the lowest levels of human capital. Country 1's, 2's, and 3's inflation policy parameters are respectively 3 percent, 4 percent and 5 percent. We term country 1 as industrialized economy (IE), country 2 as newly industrialized economy (NIE), and country developing 3 country (DE). We specify the values of the parameters, α_j, in the Cobb-Douglas productions approximately equal to 0.3 (for instance, Miles and Scott, 2005; Abel *et al*., 2007). In our specifications of the total factor productivities, we emphasize their relative values. A recent literature review of estimating A_j is provided by Delpachitra and Dai (2012). We also assume that different groups have different propensities to save and to hold money. The depreciation rate of physical capital is specified at 0.05. Corresponding to equations (17), we have

$$\bar{k}_{j1} = b_{j1} + R_{j1}\,\tilde{r}_{j1}\,\bar{k}_{j1} + R_{j1}\,\tilde{r}_{j2}\,\bar{k}_{j2},$$
$$\bar{k}_{j2} = b_{j2} + R_{j2}\,\tilde{r}_{j1}\,\bar{k}_{j1} + R_{j2}\,\tilde{r}_{j2}\,\bar{k}_{j2}, \tag{20}$$

where $b_{jq}(k_1) \equiv \overline{W}_{jq} + R_{jq}\,\hat{N}_j$. We solve (20) as follows

$$\bar{k}_{j1} = \Omega_{j1}(k_1) \equiv \frac{b_{j1}\left(1 - R_{j2}\,\tilde{r}_{j2}\right) + b_{j2}\,R_{j1}\,\tilde{r}_{j2}}{\left(1 - R_{j1}\,\tilde{r}_{j1}\right)\left(1 - R_{j2}\,\tilde{r}_{j2}\right) - R_{j1}\,\tilde{r}_{j2} R_{j2}\,\tilde{r}_{j1}},$$

$$\bar{k}_{j2} = \Omega_{j2}(k_1) \equiv \frac{\left(1 - R_{j1}\,\tilde{r}_{j1}\right) b_{j2} + b_{j1}\,R_{j2}\,\tilde{r}_{j1}}{\left(1 - R_{j1}\,\tilde{r}_{j1}\right)\left(1 - R_{j2}\,\tilde{r}_{j2}\right) - R_{j1}\,\tilde{r}_{j2} R_{j2}\,\tilde{r}_{j1}}, \quad j = 1, 2, 3. \tag{21}$$

Insert the above six equations in (18)

$$\Omega(k_1) = \sum_{j=1}^{3} \phi_j(k_1) N_j - \sum_{j=1}^{3}\sum_{q=1}^{2} \Omega_{jq}(k_1)\,\overline{N}_{jq} = 0, \tag{22}$$

in which $\phi_1(k_1) = k_1$ and

$$\phi_j(k_1) = \left(\frac{\alpha_1 A_1 k_1^{-\beta_1} - \delta_j}{\alpha_j A_j}\right)^{-1/\beta_j}, \quad j = 2, 3.$$

With the parameter values in (19), we first determine the equilibrium value of k_1 by (22). Then, following Lemma 2, we determine the equilibrium values of all the variables. As shown in Figure 1, $\Omega(k_1) = 0$ has a unique positive meaningful solution (we also check the equation for the rest range of the variables).

The equilibrium values are listed in (23).

$$F = 521.97,\ C = 471.16,\ K = 953.64,\ r = 0.128,$$

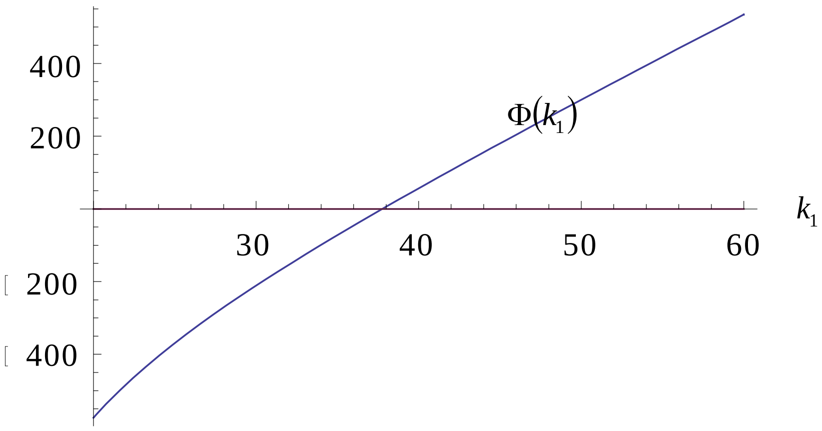
$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 37.77 \\ 21.07 \\ 13.35 \end{pmatrix}, \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 20.13 \\ 12.48 \\ 7.12 \end{pmatrix}, \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 20.28 \\ 8.48 \\ 2.28 \end{pmatrix}, \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 322.11 \\ 137.23 \\ 62.64 \end{pmatrix},$$

$$\begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 604.38 \\ 231.74 \\ 117.53 \end{pmatrix}, \begin{pmatrix} \overline{K}_1 \\ \overline{K}_2 \\ \overline{K}_3 \end{pmatrix} = \begin{pmatrix} 542{,}33 \\ 310.77 \\ 100.54 \end{pmatrix}, \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} -7.92 \\ 10.09 \\ -2.17 \end{pmatrix},$$

$$\begin{pmatrix} w_{11} \\ w_{12} \\ w_{21} \\ w_{22} \\ w_{31} \\ w_{32} \end{pmatrix} = \begin{pmatrix} 40.23 \\ 26.84 \\ 17.47 \\ 8.73 \\ 4.75 \\ 2.85 \end{pmatrix}, \quad \begin{pmatrix} \bar{k}_{11} \\ \bar{k}_{12} \\ \bar{k}_{21} \\ \bar{k}_{22} \\ \bar{k}_{31} \\ \bar{k}_{32} \end{pmatrix} = \begin{pmatrix} 139.95 \\ 52.49 \\ 66.51 \\ 22.25 \\ 13.70 \\ 5.72 \end{pmatrix}, \quad \begin{pmatrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \\ c_{31} \\ c_{32} \end{pmatrix} = \begin{pmatrix} 57.87 \\ 33.65 \\ 25.78 \\ 11.68 \\ 6.45 \\ 3.58 \end{pmatrix}, \quad \begin{pmatrix} m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \\ m_{31} \\ m_{32} \end{pmatrix} = \begin{pmatrix} 28.93 \\ 16.82 \\ 12.89 \\ 5.84 \\ 3.22 \\ 1.81 \end{pmatrix}, \tag{23}$$

in which $F \equiv \sum_{j=1}^{3} F_j$, $C \equiv \sum_{j=1}^{3} C_j$, $E_j \equiv r\left(\bar{K}_j - K_j\right)$.

Figure 1. The Unique Solution



We see that the IE's capital intensity is much higher than the NIE's and the NIE's capital intensity is much higher than the DE's. There are also great differences in wage rate and per capita wealth within and among national economies. For instance, the wage rate of household (1, 1) is almost 15 times as high as that of household (3, 2); the per capita wealth level of household (1, 1) is almost 20 times as high as that of household (3, 2). There are also great differences in terms of national output levels, per-capita output levels, per-capita consumption levels, and real money holdings. The NIE's trade is in deficit and the other two economies in surplus. We see that globalization will not lead to convergence in the long term as long as nations are different in human capital and preferences. It should be noted that here we neglect effects of possible free migration among nations upon the global economy.

5. Comparative Static Analysis

As the system has a unique equilibrium, we make comparative static analysis. As we have provided the procedure to determine the values of all the variables, it is straightforward to examine effects of changes in any parameter on the steady state. This section is concerned with how the global economy is affected as national conditions are changed. We introduce a symbol, $\bar{\Delta}$, by which a variable $\bar{\Delta} x$ stand for the change rate of the variable x in percentage due to changes in parameter value.

An improvement in the developing economy's technology

First we study effects of changes in the DE's technology on the national economy and trade patterns. In the literature of economic development and economic geography differences in technologies and human capital are considered as key determinants of spatial differences in economic growth and living standards (e.g., Grossman and Helpman, 1991; Storper and Venables, 2004; Rodriguez and Crescenzi, 2008). We now examine how a change in the total productivity in one country affects the global trade patterns and each country's economic development. We increase the DE's total productivity A_3 from 3 to 3.5. The simulation results are illustrated in (24). As the DE improves its productivity, the global output, wealth and consumption are all increased. The rate of interest is increased. The improvement in productivity of the DE improves the aggregated economic performance of the global economy. Nevertheless, when examining effects of national economies, we see that different economies are affected differently. As the DE improves its technology, not only the country's national output, capital employed, wealth and consumption are increased, but also the country's capital intensity, wage rates, per capita consumption, wealth, and real money holdings of the both groups are improved. Hence, the DE benefits from its technological improvement not only in national aggregated variables but also in all individuals' terms. As demonstrated in (24), the IE's and NIE's national output, capital employed, and capital intensities are all reduced. Moreover, the variables for individuals, wage rates, per capita consumption, wealth, and real money holdings of the both groups are either increased or reduced. We see that some variables of the developed and newly developed economies will not benefit from the technological advance of the developing economy. This occurs partly because as the DE improves its productivity, it absorbs more capital and increases the cost of capital in the global market. The increased capital cost in the global market reduces the capital intensities in the other two economies. As The NIE's trade is in deficit and the other two economies in surplus before the technological change, the IE's and NIE's trade balances are improved and the DE's trade balance is deteriorated.

$$A_3: 3 \Rightarrow 3.5,\ \overline{\Delta} F = 2.98,\ \overline{\Delta} C = 2.99,\ \overline{\Delta} K = 2.75,\ \overline{\Delta} r = 0.41,$$

$$\begin{pmatrix} \overline{\Delta} k_1 \\ \overline{\Delta} k_2 \\ \overline{\Delta} k_3 \end{pmatrix} = \begin{pmatrix} -0.44 \\ -0.44 \\ 25.46 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} f_1 \\ \overline{\Delta} f_2 \\ \overline{\Delta} f_3 \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.13 \\ 25.83 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} m_1 \\ \overline{\Delta} m_2 \\ \overline{\Delta} m_3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.05 \\ 25.99 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} F_1 \\ \overline{\Delta} F_2 \\ \overline{\Delta} F_3 \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.13 \\ 25.83 \end{pmatrix},$$

$$\begin{pmatrix} \overline{\Delta} K_1 \\ \overline{\Delta} K_2 \\ \overline{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -0.44 \\ -0.42 \\ 25.46 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} \bar{K}_1 \\ \overline{\Delta} \bar{K}_2 \\ \overline{\Delta} \bar{K}_3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.05 \\ 25.99 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} E_1 \\ \overline{\Delta} E_2 \\ \overline{\Delta} E_3 \end{pmatrix} = \begin{pmatrix} -3.88 \\ 1.85 \\ 22.79 \end{pmatrix},$$

$$\begin{pmatrix} \overline{\Delta} w_{11} \\ \overline{\Delta} w_{12} \\ \overline{\Delta} w_{21} \\ \overline{\Delta} w_{22} \\ \overline{\Delta} w_{31} \\ \overline{\Delta} w_{32} \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.15 \\ -0.13 \\ -0.13 \\ 25.83 \\ 25.83 \end{pmatrix},\ \begin{pmatrix} \overline{\Delta} \bar{k}_{11} \\ \overline{\Delta} \bar{k}_{12} \\ \overline{\Delta} \bar{k}_{21} \\ \overline{\Delta} \bar{k}_{22} \\ \overline{\Delta} \bar{k}_{31} \\ \overline{\Delta} \bar{k}_{32} \end{pmatrix} = \begin{pmatrix} \overline{\Delta} c_{11} \\ \overline{\Delta} c_{12} \\ \overline{\Delta} c_{21} \\ \overline{\Delta} c_{22} \\ \overline{\Delta} c_{31} \\ \overline{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} \overline{\Delta} m_{11} \\ \overline{\Delta} m_{12} \\ \overline{\Delta} m_{21} \\ \overline{\Delta} m_{22} \\ \overline{\Delta} m_{31} \\ \overline{\Delta} m_{32} \end{pmatrix} = \begin{pmatrix} 0.03 \\ -0.04 \\ 0.07 \\ 0.01 \\ 26.02 \\ 25.95 \end{pmatrix}. \qquad (24)$$

A rise in in the industrialized economy's inflation policy

This section is concerned with effects of changes in some parameters on the national economy and regional economic structures. First, we are concerned with the inflation policy. Modern analysis of the long-term interaction of inflation and capital formation begins with Tobin's seminal contribution (Tobin, 1965; McCallum, 1983; Walsh, 2003). Tobin showed that an increase in the level of the inflation rate will increase the capital stock of an economy. It should be noted that there are only a few formal monetary growth models with internal trade are proposed in the literature of international economics. We now raise the IE's inflation policy as follows, $\mu_1 : 0.03 \Rightarrow 0.05$. The results are listed in (25). The rise in the inflation policy enhances the equilibrium values of the global output, consumption level and physical wealth, but reduces the rate of interest. The IE's trade balance is improved and the DE's and NIE's trade balances are deteriorated. The effects of the change on the other variables are given in (25). It should be remarked that although the country which raises its inflation policy benefits in every aspect, the other countries suffer in some aspects and benefit in others. This also implies that if not only one country changes its monetary policy, the global effects of printing more money on different countries have ambiguous effects, except on the country which speeds up printing money. In this study, we don't introduce endogenous mechanism for determining speed of printing money. In globally well-connected economies different countries will react differently when one country initiates speeding up money. Moreover, one referee points out, "A higher domestic inflation rate intuitively discourages domestic real money holdings. The resulting rise in the transactions cost lowers the marginal product of capital and thereby suppresses private investment and thus the rate of economic growth. The reduction in domestic real money holdings causes the nominal rate of interest to rise and leads domestic residents to hold foreign currencies." For simplicity of analysis, our model is limited to the case that money is held only by the domestic residents. It is more realistic to allow foreigners to hold money.

$$\mu_1 : 0.03 \Rightarrow 0.05, \ \bar{\Delta} F = 0.57, \ \bar{\Delta} C = 0.45, \ \bar{\Delta} K = 1.78, \ \bar{\Delta} r = -1.65,$$

$$\begin{pmatrix} \bar{\Delta} k_1 \\ \bar{\Delta} k_2 \\ \bar{\Delta} k_3 \end{pmatrix} = \begin{pmatrix} 1.80 \\ 1.72 \\ 1.80 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} f_1 \\ \bar{\Delta} f_2 \\ \bar{\Delta} f_3 \end{pmatrix} = \begin{pmatrix} 0.60 \\ 0.51 \\ 0.60 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} m_1 \\ \bar{\Delta} m_2 \\ \bar{\Delta} m_3 \end{pmatrix} = \begin{pmatrix} 0.82 \\ -0.18 \\ 0.09 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} F_1 \\ \bar{\Delta} F_2 \\ \bar{\Delta} F_3 \end{pmatrix} = \begin{pmatrix} 0.60 \\ 0.51 \\ 0.60 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} 1.80 \\ 1.71 \\ 1.80 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} \bar{K}_1 \\ \bar{\Delta} \bar{K}_2 \\ \bar{\Delta} \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 3.23 \\ -0.19 \\ 0.07 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} E_1 \\ \bar{\Delta} E_2 \\ \bar{\Delta} E_3 \end{pmatrix} = \begin{pmatrix} -12.14 \\ -7.34 \\ 10.18 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} w_{11} \\ \bar{\Delta} w_{12} \\ \bar{\Delta} w_{21} \\ \bar{\Delta} w_{22} \\ \bar{\Delta} w_{31} \\ \bar{\Delta} w_{32} \end{pmatrix} = \begin{pmatrix} 0.60 \\ 0.60 \\ 0.51 \\ 0.51 \\ 0.60 \\ 0.60 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} \bar{k}_{11} \\ \bar{\Delta} \bar{k}_{12} \\ \bar{\Delta} \bar{k}_{21} \\ \bar{\Delta} \bar{k}_{22} \\ \bar{\Delta} \bar{k}_{31} \\ \bar{\Delta} \bar{k}_{32} \end{pmatrix} = \begin{pmatrix} 2.82 \\ 3.66 \\ -0.29 \\ -0.03 \\ -0.01 \\ 0.17 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{12} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{31} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} \bar{\Delta} m_{11} \\ \bar{\Delta} m_{12} \\ \bar{\Delta} m_{21} \\ \bar{\Delta} m_{22} \\ \bar{\Delta} m_{31} \\ \bar{\Delta} m_{32} \end{pmatrix} = \begin{pmatrix} 0.47 \\ 1.07 \\ -0.29 \\ -0.03 \\ -0.01 \\ 0.17 \end{pmatrix}. \tag{25}$$

A rise in the industrialized economy's propensity to save

We now raise household (1, 1)'s propensity as follows: $\lambda_{011}: 0.85 \Rightarrow 0.9$. The effects are listed in (26). As the propensity to save falls, as the neoclassical growth theory predicts, the rate of interest is reduced. The capital intensities, output levels, wage rates, and consumption levels are all enhanced. The wealth per household of the rich group in the developed economy is increased. The wealth levels of the two groups in the NIE are lessened. Although the NIE's wage rates and output are increased, the per capita wealth, consumption level and money holding are reduced. Hence, the NIE suffers from the rise in the IE's propensity to save. In the DE the rich group suffers but the poor group benefits from the preference change. The effects of the change on the other variables are given in (26). Country 1 benefits in every aspect by increasing the rich households' propensity to save. Nevertheless, this change has negative effects on some variables in the other countries

$$\lambda_{011}: 0.85 \Rightarrow 0.9, \ \bar{\Delta} F = 0.96, \ \bar{\Delta} C = 0.76, \ \bar{\Delta} K = 2.99, \ \bar{\Delta} r = -2.74,$$

$$\begin{pmatrix} \bar{\Delta} k_1 \\ \bar{\Delta} k_2 \\ \bar{\Delta} k_3 \end{pmatrix} = \begin{pmatrix} 3.03 \\ 2.88 \\ 3.03 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} f_1 \\ \bar{\Delta} f_2 \\ \bar{\Delta} f_3 \end{pmatrix} = \begin{pmatrix} \bar{\Delta} F_1 \\ \bar{\Delta} F_2 \\ \bar{\Delta} F_3 \end{pmatrix} = \begin{pmatrix} 1.00 \\ 0.86 \\ 1.00 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} m_1 \\ \bar{\Delta} m_2 \\ \bar{\Delta} m_3 \end{pmatrix} = \begin{pmatrix} 1.37 \\ -0.28 \\ 0.15 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} 3.03 \\ 2.88 \\ 3.03 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} \bar{K}_1 \\ \bar{\Delta} \bar{K}_2 \\ \bar{\Delta} \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 5.42 \\ -0.32 \\ 0.13 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} E_1 \\ \bar{\Delta} E_2 \\ \bar{\Delta} E_3 \end{pmatrix} = \begin{pmatrix} -20.11 \\ -12.16 \\ 16.89 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} w_{11} \\ \bar{\Delta} w_{12} \\ \bar{\Delta} w_{21} \\ \bar{\Delta} w_{22} \\ \bar{\Delta} w_{31} \\ \bar{\Delta} w_{32} \end{pmatrix} = \begin{pmatrix} 1.00 \\ 1.00 \\ 0.86 \\ 0.86 \\ 1.00 \\ 1.00 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} \bar{k}_{11} \\ \bar{\Delta} \bar{k}_{12} \\ \bar{\Delta} \bar{k}_{21} \\ \bar{\Delta} \bar{k}_{22} \\ \bar{\Delta} \bar{k}_{31} \\ \bar{\Delta} \bar{k}_{32} \end{pmatrix} = \begin{pmatrix} 10.18 \\ 0.34 \\ -0.47 \\ -0.04 \\ -0.01 \\ 0.29 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{12} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{31} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} \bar{\Delta} m_{11} \\ \bar{\Delta} m_{12} \\ \bar{\Delta} m_{21} \\ \bar{\Delta} m_{22} \\ \bar{\Delta} m_{31} \\ \bar{\Delta} m_{32} \end{pmatrix} = \begin{pmatrix} 2.88 \\ 0.34 \\ -0.47 \\ -0.04 \\ -0.01 \\ 0.29 \end{pmatrix}. \tag{26}$$

6. Conclusions

This paper proposed a multi-country growth model with heterogeneous groups in each country and endogenous wealth accumulation. We show that the dynamics of the world economy is controlled by a set of differential equations. We also simulated the model with the Cobb-Douglas production functions and demonstrated effects of changes in some parameters. We show that different economies may react differently to these changes. As we have explicitly shown the computational procedure, we can simulate the world economy with any number of economies and any types of households. This paper examined the equilibrium behavior of a three-country world economy with two groups in each country. We examined, for instance, as the IE's inflation policy is increased, the equilibrium values of the global output, consumption level and physical wealth are enhanced, but the rate of interest lowered. The IE's trade balance is improved and the DE's and NIE's trade balances are deteriorated. The country which raises its inflation policy benefits in every aspect, the other countries suffer in some aspects and benefit in others. Practically, this also implies that if one country speeds up printing money, other countries in the well-connected global economy may also speed up printing money. It should be noted that our conclusion is obtained on the assumption that only domestic households hold domestic money. If we allow any household in the global economy may hold money of any economy, our conclusion may be different. Our analysis on the impact of preference change also provides important insights into complexity of globally interconnected world economies. For instance, it is well known that the USA economy has low saving rates. If one group of the USA increases its propensity to save, the living conditions of all the groups and USA economy are improved, even though the effects on some groups and some economies may not be beneficial. Since our analytically tractable framework is based on microeconomic foundation and treats the global economy as a connected whole, it may enable us to analyze other important issues. It is possible to extend the model in different directions. For instance, we may consider that each economy has multiple sectors. Another important direction to generalize the study is to take account of changeable returns to scale in different economies. An old question in monetary economics is how to analyze situation-dependent monetary policies (e.g., Cavalcanti and Nosal, 2009).

Appendix: Proving Lemma 1

First, from equations (1) we obtain $f_j'(k_j) = f_1'(k_1) - \delta_j,\ j \equiv 2, \cdots, J$, where $\delta_j \equiv \delta_{k1} - \delta_{kj}$. If $f_1'(k_1) - \delta_j > 0$ for all $j = 2, \cdots, J$ and given $k_1(t) > 0$, then the equations determine unique relations between k_j and k_1, denoted by $k_j = \phi_j(k_1),\ j = 1, \cdots, J$, where $\phi_1(k_1) = k_1$. From equations $f_j'(k_j) = f_1'(k_1) - \delta_j$, we have $f_j''(k_j) dk_j / dk_1 = f_1''(k_1),\ j = 2, \cdots, J$. As $f_j''(k_j) \le 0,\ j = 1, \cdots, J$, we see that $dk_j / dk_1 \ge 0,\ j = 2, \cdots, J$. That is, $\phi_j'(k_1) \ge 0$. Hence, for any given $k_1(t) > 0$, we determine $k_j(t)$ as unique functions of $k_1(t)$. From equations (1), we determine the wage rates as functions of $k_1(t)$ as follows

$$w_{jq}(t) = \bar{\phi}_{jq}(k_1) \equiv h_{jq}\,\bar{\phi}_j(k_1),\quad w_j(t) = \bar{\phi}_j(k_1) \equiv f_j(\phi_j(k_1)) - \phi_j(k_1) f_j'(\phi_j(k_1)),\quad j = 1, \cdots, J. \tag{A1}$$

We can rewrite (8) as

$$\sum_{j=1}^{J} k_j N_j = \sum_{j=1}^{J} \sum_{q=1}^{Q_j} \bar{k}_{jq} \bar{N}_{jq}.$$

Insert equation $k_j = \phi_j(k_1)$ into the above equation

$$\bar{k}_{11} = \Lambda(k_1, \{\bar{k}\}) \equiv \sum_{j=1}^{J} n_j \phi_j - \sum_{j=2}^{J} \sum_{q=1}^{Q_j} n_{jq} \bar{k}_{jq} - \sum_{q=2}^{Q_1} n_{1q} \bar{k}_{1q}, \tag{A2}$$

in which $n_j \equiv \dfrac{N_j}{\bar{N}_{11}},\ n_{jq} \equiv \dfrac{\bar{N}_j}{\bar{N}_{11}},\ \{\bar{k}\} \equiv (\bar{k}_{12}, \cdots, \bar{k}_{JQ_J})$.

We see that household $(1, 1)$'s per capita physical wealth, $\bar{k}_{11}(t)$, can be expressed as a unique function of country 1's capital intensity and the other countries' per capita physical wealth $\{\bar{k}(t)\}$ at any point of time. From equations (1) and (A2) and the definitions of $\bar{y}_j$, we have

$$\bar{y}_{11}(t) = \Lambda_{11}(k_1, \{\bar{k}\}, m_1) \equiv \phi_r(k_1) \Lambda(k_1, \{\bar{k}\}) + \bar{\phi}_{11}(k_1) + \mu_1 m_1,$$

$$\bar{y}_{jq}(t) = \Lambda_{jq}\left(k_1, \{\bar{k}\}, m_j\right) \equiv \phi_r(k_1)\bar{k}_{jq} + \bar{\phi}_{jq}(k_1) + \mu_j m_j,$$

$$j = 1, \ldots J, \quad q = 1, \ldots, Q_j, \quad (j, q) \neq (1, 1), \tag{A3}$$

where $\phi_r(k_1) \equiv 1 + f_1^{'}(k_1) - \delta_{k1}$. From $m_{jq} = \chi_{jq} c_{jq}$, $\left(\bar{\chi}_{jq} + \chi_{jq}\pi_j\right)c_{jq} = \xi_{jq}\bar{y}_{jq}$, and (9), we solve

$$\bar{N}_j m_j = \sum_{q=1}^{Q_j} \frac{\xi_{jq}\,\chi_{jq}\,\bar{N}_{jq}\,\bar{y}_{jq}}{\bar{\chi}_{jq} + \chi_{jq}\pi_j}. \tag{A4}$$

As we want to express inflation rates as functions of the other variables by (A4) and it is difficult to do this, for simplicity of analysis we assume that all the households within a country have the equal rate of χ_{jq}, that is, $\chi_j = \chi_{jq}$ ($\bar{\chi}_j = \bar{\chi}_{jq}$). Under this assumption from (A4) we solve

$$\pi_j = \bar{\Lambda}_j\left(k_1, \{\bar{k}\}, m_j\right) \equiv \left(\frac{1}{\bar{N}_j\, m_j}\sum_{q=1}^{Q_j}\xi_{jq}\,\bar{N}_{jq}\,\bar{y}_{jq}\right) - \frac{\bar{\chi}_j}{\chi_j}. \tag{A5}$$

From $m_{jq} = \chi_j c_{jq}$, $\left(\bar{\chi}_j + \chi_j\pi_j\right)c_{jq} = \xi_{jq}\bar{y}_{jq}$, and (A5), we solve

$$m_{jq} = \hat{\Lambda}_{jq}\left(k_1, \{\bar{k}\}, m_j\right) \equiv \frac{\chi_j\,\xi_{jq}\,\Lambda_{jq}\left(k_1, \{\bar{k}\}, m_j\right)}{\bar{\chi}_j + \chi_j\bar{\Lambda}_j\left(k_1, \{\bar{k}\}, m_j\right)}. \tag{A6}$$

Equations (A5) show that a country's inflation rate is function of the global distribution of capital stocks and its real money per capita. Substituting (A5) into (7) yields

$$\dot{m}_j = \Psi_j\left(k_1, \{\bar{k}\}, m_j\right) \equiv \left(\mu_j - \bar{\Lambda}_j\left(k_1, \{\bar{k}\}, m_j\right)\right)m_j. \tag{A7}$$

Insert $s_{jq} = \lambda_{jq}\bar{y}_{jq}$ and (A3) in (6)

$$\dot{\bar{k}}_{11} + \dot{m}_{11} = \lambda_{11}\Lambda_{jq}\left(k_1, \{\bar{k}\}, m_1\right) - \bar{k}_{11} - \hat{\Lambda}_{11}\left(k_1, \{\bar{k}\}, m_1\right),$$

$$\dot{\bar{k}}_{jq} + \dot{m}_{jq} = \bar{\Psi}_{jq}\left(k_1, \{\bar{k}\}, m_j\right) \equiv \lambda_{jq}\Lambda_{jq}\left(k_1, \{\bar{k}\}, m_j\right) - \bar{k}_{jq} - \hat{\Lambda}_{jq}\left(k_1, \{\bar{k}\}, m_j\right),$$

$$j = 1, \ldots., J, \quad q = 1, \ldots, Q_j, \quad (j, q) \neq (1, 1), \tag{A8}$$

where we also use (A6). Taking derivatives of (A2) with respect to time yields

$$\dot{\bar{k}}_{11} = \left[\sum_{j=1}^{J} n_j\phi_j^{'}\right]\dot{k}_1 - \sum_{j=2}^{J}\sum_{q=1}^{Q_j} n_{jq}\dot{\bar{k}}_{jq} - \sum_{q=2}^{Q_1} n_{1q}\dot{\bar{k}}_{1q}. \tag{A9}$$

Insert (A9) in the first equation in (A8)

$$\left[\sum_{j=1}^{J} n_j \phi_j^{'}\right]\dot{k}_1 - \sum_{j=2}^{J}\sum_{q=1}^{Q_j} n_{jq}\dot{\bar{k}}_{jq} - \sum_{q=2}^{Q_1} n_{1q}\dot{\bar{k}}_{1q} + \dot{m}_{11} = \lambda_{11}\Lambda_{jq}\left(k_1, \{\bar{k}\}, m_1\right) - \bar{k}_{11} - \hat{\Lambda}_{11}\left(k_1, \{\bar{k}\}, m_1\right).$$

(A10)

On the other hand, taking derivatives of (A6) with respect to time yields

$$\dot{m}_{jq} = \frac{\partial \hat{\Lambda}_{jq}}{\partial k_1}\dot{k}_1 + \frac{\partial \hat{\Lambda}_{jq}}{\partial m_j}\dot{m}_j - \sum_{i=2}^{J}\sum_{p=1}^{Q_j}\frac{\partial \hat{\Lambda}_{jq}}{\partial \bar{k}_{ip}} - \sum_{p=2}^{Q_1} n_{1p}\dot{\bar{k}}_{1p}. \quad \text{(A11)}$$

Substituting (A11) into (A10) and (A8), we assume that we can solve the resulted linear (in the derivatives) equations as (10). In summary, we obtain Lemma 1.

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